A-LEVEL
MATHEMATICS

MS2B – Statistics 2B
Mark scheme

6360
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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks.**

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Marks	Total	Comments
	Throughout parts **(a)** to **(d)**			Unsupported correct answers score full marks
(a)	0.703	B1	**1**	AWRT
(b)	$e^{-2.3} + e^{-2.3} \times 2.3$ (= 0.100… + 0.231…) (= 0.331) $1 - (e^{-2.3} + e^{-2.3} \times 2.3) = 0.669$	M1 A1 A1	**3**	PI For either P(X=0) or P(X=1) For both P(X=0) and P(X=1) or sum AWRT 0.669
(c)	**Use** of Po(6) 0.9799 for top value - 0.8472 for bottom value **Or** $e^{-6}\left(\frac{6^9}{9!}+\frac{6^{10}}{10!}+\frac{6^{11}}{11!}\right)$ = 0.1327 = 0.133	M1 A1 A1	**3**	0.7440, 0.8472, 0.9161, 0.9799, 0.9912 or 0.9964 seen AWFW 0.9798 to 0.9800 AWFW 0.8470 to 0.8473 A1 for **either** of these values or this expression OE AWRT 0.133
(d)	**Use** of Po(0.8) $e^{-0.8} \times 0.8^2/2$ or 0.9526 – 0.8088 = 0.1438 = 0.144	M1 m1 A1	**3**	Correct formula or 0.9526 or 0.8088 seen AWRT 0.144
(e)	$(1 - 0.0111) \times (1 - e^{-2.3}) \times (1 - 0.3012)$ $0.9889 \times 0.8997 \times 0.6988 = 0.622$	B1 M1 A1	**3**	For any one of these seen or PI by the correct value seen (3 sf or better) For all three correct and multiplied PI AWRT
			13	

Q2	Solution	Marks	Total	Comments
	$\bar{x}$ or μ = (32.93 + 30.47) ÷ 2 = 31.7	B1		CAO $\bar{x}$ or μ not necessary but do not ISW. If contradictory value for μ seen then B0.
	t_9 = 2.262	M1		AWRT 2.26
	(32.93 – 30.47) = (2 × 2.262 × s) ÷ √10	m1		OE single correct equation with only s or σ unknown
	s = 1.72 so unbiased estimate for σ^2 is 2.96	A1	**4**	AWFW 2.95 to 2.96 Final answer 1.72 earns M1 m1 A0
			4	

Q3	Solution	Marks	Total	Comments
(a)	0.35	B1	**1**	CAO or equivalent fraction or %
(b)	P(< 3) = 0.45 '0.35' × '0.45' (= 0.1575) × 2 = 0.315	B1 M1 A1	**3**	 Their 0.35 and 0.45 CAO or equivalent fraction or %
(c)	Mean = 1 × 0.19 + 2 × 0.26 + 3 × 0.20 + 4 × 0.13 + 5 × 0.07 + 6 × 0.15 = 0.19 + 0.52 + 0.60 + 0.52 + 0.35 + 0.90 = 3.08 Variance = 1^2 × 0.19 + 2^2 × 0.26 + 3^2 × 0.20 + 4^2 × 0.13 + 5^2 × 0.07 + 6^2 × 0.15 – 3.08^2 = 2.77(36)	M1 A1 M1 A1	**4**	This working, or the next line, must be seen (at least 3 products) CAO. AG. PI AWFW 2.77 to 2.78
(d)	No probability of 0 books borrowed.. ..and no probability of more than 6 books. Wrong shape or probability increases at top end.	 E2,1,0	**2**	Cannot borrow no books Cannot borrow more than 6 books E1 for each of these 3 distinct points up to a maximum of 2 marks
(e)(i) **(ii)**	10 × 3.08 = 30.8 (pence) '2.7736' × 100 = '277.36' then √'277.36' = 16.7 (pence) **or** √'2.7736' = '1.66… ', '1.67' × 10 = 16.7 (pence) Where working is in £ in (i) or (ii) or both (i) £0.308 (ii) '2.7736' × 0.01 = 0.027736 √'0.027736' = £0.167 **Or** √'2.7736' = '1.66... ', '1.67' × 0.1 = £0.167	B1 M1 A1 (M1) (A1) (B1) (M1) (A1) (M1) (A1)	**3**	CAO 31, without 30.8 seen, scores B0 For their variance × 100 and √ AWFW 16 to 17 For √ their variance and × 10 AWFW 16 to 17 Must show £ sign For their variance × 0.01 and √ AWFW 0.16 to 0.17. Must show £ sign For √ their variance and × 0.1 AWFW 0.16 to 0.17. Must show £ sign
			13	

Q4	Solution	Marks	Total	Comments
(a)	10	B1	**1**	CAO
(b)	0.07 × 10 = 0.7	B1	**1**	OE
(c)(i)	$E(X) = {}^{1}/_{20}$	B1		Decimal or fraction
(ii)	$E(X^2) = \int_0^{0.1} 10x^2 \mathrm{d}x$	M1		Integration, correct limits, their *k*
	$= [{}^{10}/_{3}\, x^3]_0^{0.1} = {}^{1}/_{300}$	A1		Or at least 3sf equivalent
(iii)	$\mathrm{Var}(X) = E(X^2) - E(X)^2 = {}^{1}/_{300} - ({}^{1}/_{20})^2$	M1		Correct expression – allow use of 0.003 or 0.0033 or better for M1
	$= {}^{1}/_{1200}$ So sd = $\sqrt{({}^{1}/_{1200})} = {}^{\sqrt{3}}/_{60} = 0.0289$	A1	**5**	Equivalent surd or AWFW 0.0288 to 0.0289 Correct answer but not derived from **(ii)** scores M0 B1.
			7	

Q5	Solution	Marks	Total	Comments
(a)	Expected values **<3** | **<12** | **None** | **Total** **Petrol** | 12.04 | 36.98 | 165.98 | 215 **Diesel** | 1.96 | 6.02 | 27.02 | 35 **Total** | 14 | 43 | 193 | 250	M1		For attempt to find expected values (at least 2 correct) either before or after combining. Allow rounding to nearest integer or 1 d.p.
	Combine first two columns Observed **Problems** | **None** | **Total** **Petrol** | 45 | 170 | 215 **Diesel** | 12 | 23 | 35 **Total** | 57 | 193 | 250	M1 A1		For combining (at least 1 correct) Combined Observed CAO
	Expected **Problems** | **None** | **Total** **Petrol** | **49.02** | **165.98** | **215** **Diesel** | **7.98** | **27.02** | **35** **Total** | **57** | **193** | **250**	A1		Combined expected (both correct). Allow rounding to nearest integer or 1 d.p.
	$\lvert(O - E)\rvert - 0.5 = 3.52$ $\frac{3.52^2}{49.02} + \frac{3.52^2}{7.98} + \frac{3.52^2}{165.98} + \frac{3.52^2}{27.02}$ $(0.2527 + 1.5526 + 0.0746 + 0.4585)$ $= 2.338\ldots$	M1 A1 B1		Attempt including Yates (at least 1 correct) AWRT 2.34 (Implies first 6 marks) PI by CV
	1 degree of freedom CV = 2.706	B1 B1		AWRT 2.71 Allow "problems independent of engine type" but not reverse or "they are independent" or simply "no association"
	H_0: No association between problems and engine type			
	$2.34 < 2.706$ (so accept H_0)	A1 dep		Comparison or a diagram equivalent Dep on A1 for ts and B1 for CV
	There is no significant evidence of an association between mechanical problems and type of engine.	E1 dep		OE Conclusion in context. Dep on previous A1
			11	
(b)	Dependent on a conclusion of **no association**. No evidence that diesels have more problems	E1		Even from incorrect working Use of result, however any additional suggestion that diesels may have more problems then E0
	Go with her preference or buy either.	E1 dep		Or similar dep on previous E1
			2	
			13	

No Yates: Can score M1, M1, A1, A1, M0, A0, B1, B1, B1, A0, E0 = 7 out of 11 ($\chi^2 = 3.05$)

No combining: Can score M1, M0, A0, A0, M0, A0, B1, B1 (for 2 d.o.f. giving 4.605), B1, A0, E0 = 4 out of 11 ($\chi^2 = 3.92$)

Just 2 cells combined (1.96 and 6.02) can score M1 M1 and B1 for hypotheses

Q6	Solution	Marks	Total	Comments
(a)	H_0: $\mu = 18.2$ (or $\mu \geq 18.2$) H_1: $\mu < 18.2$	B1		For both. μ or "population mean"
	$\bar{x} = 1384.5 \div 78 = 17.75$	B1		CAO
	test stat = $\dfrac{\text{'17.75'} - 18.2}{(1.8 \div \sqrt{78})}$ $= -2.208$	M1 A1		Condone 18.2 – '17.75' for M1 AWRT –2.21 Must be **negative**
	$z_{crit} = \pm 2.0537$	B1		AWRT ±2.05
	$-2.208 < -2.0537$ or $2.208 > 2.0537$ in critical region, reject H_0 or accept H_1	A1 dep		Comparison stated or diagram Dep on previous A1 and B1
	There is significant evidence at the 2% level of significance to support Gerald's belief.	E1 dep	**7**	**Context** conclusion. Dep on A1 dep. Definitive conclusions (Eg Gerald is correct) score E0. If not referring to belief, must use "mean" or "average"
	Alternative Calculation of critical region boundary value **or** using confidence interval			
	H_0 and H_1 as above	(B1)		
	$\bar{x} = 1384.5 \div 78 = 17.75$	(B1)		CAO.
	$z_{crit} = \pm 2.0537$	(B1)		AWRT ±2.05
	$18.2 - 2.0537 \times \dfrac{1.8}{\sqrt{78}}$ | $17.75 + 2.0537 \times \dfrac{1.8}{\sqrt{78}}$	(M1)		Must be subtracting/adding as appropriate
	$= 17.78$ | $= 18.17$	(A1)		AWRT
	$17.75 < 17.78$ | $18.2 > 18.17$ so reject H_0 | so 18.2 above CI	(A1dep)		Comparison stated or diagram. Dep on previous A1
	Context conclusion as above	(E1dep)		Dep on A1 dep
(b) (i)	$\bar{x} = 16.7$, s = 1.94(3)	B1		For both, CAO & AWRT 1.94 PI
	$t_6 = 2.447$	B1		AWFW 2.44 to 2.45
	$16.7 \pm 2.447 \times \text{'1.94'} / \sqrt{7}$	M1 m1		Use of $\sqrt{7}$ Their 1.94 & rest of formula correct
	$= 14.9, 18.5$ or 16.7 ± 1.8	A1		AWRT 14.9, 18.5. Correct answer seen, no working shown scores all 5 marks.
(ii)	**18.2** (or the **mean for mainland lizards**) lies within this confidence interval so no evidence to support Gerald's belief.	A1FT E1 dep	**7**	FT provided both M marks earned and CI includes 18.2 Must be a clear statement of this. Dep on A1 FT
(c)	$\bar{x} = 19.73$ which is > 18.2 so cannot provide evidence to support Gerald's belief.	B1 E1 dep	**2**	Calculation and this comparison Accept 19.73 > mean on mainland Comment in context. Dep on B1
			16	

Q7	Solution	Marks	Total	Comments
(a)	Candidate's own grid used is acceptable.			Accept wobbly lines as straight if it seems candidate has no ruler
	Nothing drawn beyond 1 and 6 (or line at 0)	B1		Must be at least one line drawn somewhere on the diagram
	Differentiate to give f(x) = ¼ for 1 ≤ x < 4	M1		Allow this M1 if seen in (b) PI by correct line
	Straight line joining (1, ¼) to (4, ¼)	A1		Candidate's choice of scale. BOD this horizontal line.
	f(x) = $^1/_8$(6 – x) for 4 ≤ x ≤ 6	M1		OE Allow this M1 if seen in (b) PI by correct line
	Straight line joining (4, ¼) to (6, 0)	A1	**5**	
(b)	$\int_1^4 \frac{1}{4}x\,\mathrm{d}x$ and $\int_4^6 \frac{1}{8}x(6-x)\mathrm{d}x$	M1 A1		For **both** integrands correct Including correct limits for both
	E(X) = $[\frac{x^2}{8}]_1^4 + [\frac{3x^2}{8} - \frac{x^3}{24}]_4^6$	A1		Integrations done correctly and added
	= $3^1/_{24}$ or 73/24	A1	**4**	Or AWRT 3.04
			9	

